

# Math Cookbook

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August 5, 2023

## 1 Linear Algebra

**Sherman–Morrison formula:**

$$(A + uv^\top)^{-1} = A^{-1} - \frac{A^{-1}uv^\top A^{-1}}{1 + v^\top A^{-1}u} \quad (1)$$

$$(A + UCV)^{-1} = A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1} \quad (2)$$

## 2 Caculus

**Leibniz integral rule:**

$$\frac{d}{dx} \left( \int_{a(x)}^{b(x)} f(x, t) dt \right) = f(x, b(x)) \cdot \frac{d}{dx} b(x) - f(x, a(x)) \cdot \frac{d}{dx} a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, t) dt \quad (3)$$

**Grönwall's inequality** from two useful references [1, 2].

$$f(x) \geq a(x) + b(x) \int_{\underline{x}}^x f(u) du, \quad x \in [\underline{x}, \bar{x}] \quad (4)$$

$$\implies f(x) \geq a(x) + b(x) \int_{\underline{x}}^x a(u) \exp \left( \int_u^x b(s) ds \right) du, \quad x \in [\underline{x}, \bar{x}] \quad (5)$$

**Grönwall's inequality in differential form:**

$$f'(x) \leq b(x)f(x) \quad (6)$$

$$\implies f(x) \leq f(\underline{x}) \exp \left( \int_{\underline{x}}^x b(s) ds \right) \quad (7)$$

## References

- [1] G. Stephen Jones. Fundamental Inequalities for Discrete and Discontinuous Functional Equations. *Journal of the Society for Industrial and Applied Mathematics*, 12(1):43–57, 1964.

- [2] Dragoslav S. Mitrinovic, Josip Pecaric, and Arlington M. Fink. *Inequalities Involving Functions and Their Integrals and Derivatives*, volume 53. Springer Science & Business Media, 2012.